Secure Multi-Party Computation: Millionaires' Problem, Coin Tossing Problem and Federated Learning

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1. Coin Tossing Problem

2. Millionaires’ Problem

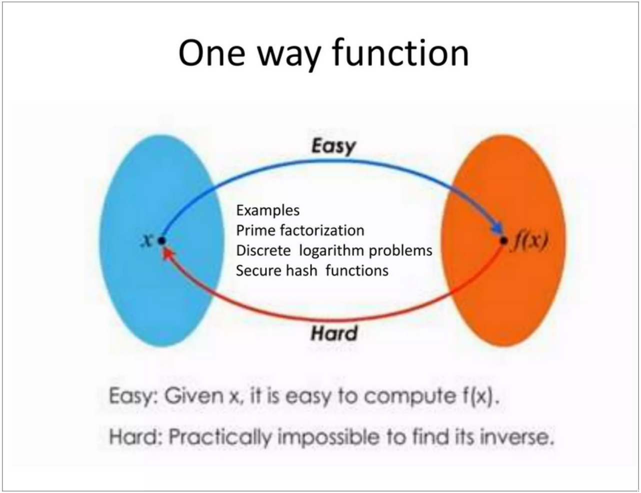
3. Federated Learning

Let's begin with a simple problem...

Consider that I'm making a bet with Junyu over the phone: let's say a bet of $100, and then we decide to use, say, a coin flip to decide who should pay who the $100. Here's the problem: how do we flip this coin?

The problem can be rephrased: how do we (2 parties) generate a unbiased random bit?
Original post by Manuel Blum[Blu83]. (Turing Award, 1995)

One way function



One way function

Examples
Prime factorization
Discrete logarithm problems
Secure hash functions

Easy: Given x, it is easy to compute f(x).

Hard: Practically impossible to find its inverse.

Figure: One way function

Solution by Blum[Blu83]

1. A flip coin: 0/1
2. choose a very big number: S, 0 -> odd / 1 -> even
3. A send to B H(S)
4. B decide to flip or not to flip it
5. A send S to B, proof H(S)

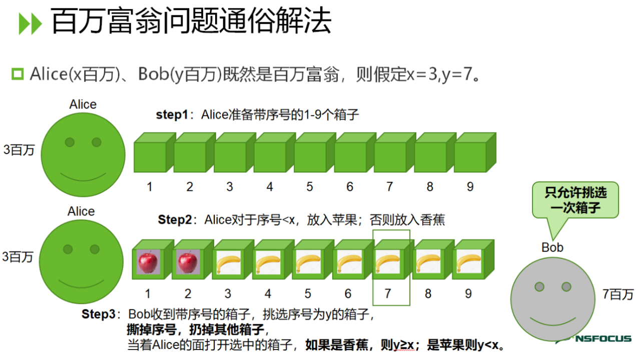
Another simple problem

Consider that I'm currently competing with Junyu to see who has more money, but neither of us wants anyone to know how much we have. (Though the reality is probably that we're comparing who has less money)

The question can be rephrased as: determine whether the inequality $a \geq b$ is true or false without revealing the actual values of a a and b.

Original post by Andrew Yao[Yao82]. (Turing Award, 2000)

A simple solution



百万富翁问题通俗解法

□ Alice(x百万)、Bob(y百万)既然是百万富翁，则假定x=3,y=7。

step1：Alice准备带序号的1-9个箱子

Step2：Alice对于序号<x，放入苹果；否则放入香蕉

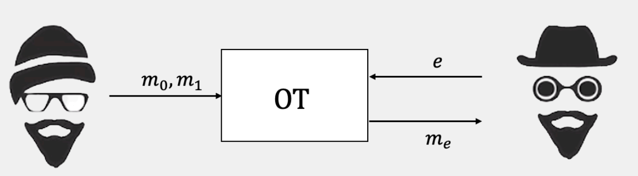
Step3：Bob收到带序号的箱子，挑选序号为y的箱子，**撕掉序号，扔掉其他箱子**，当着Alice的面打开选中的箱子，**如果是香蕉，则y≥x；是苹果则y<x。**

只允许挑选一次箱子

Figure: solution, but not rigorous

How to guarantee Bob will drop other box?
work in face-to-face situation, but not online

Oblivious Transfer



Question: How to construct a 1-out-of-N OT from 1-out-of-2 OT?

Trivial method: *cela va sans dire*
Non-trivial method: O(logn) by [NP99]

Construct 1-out-of-2 OT by trapdoor function

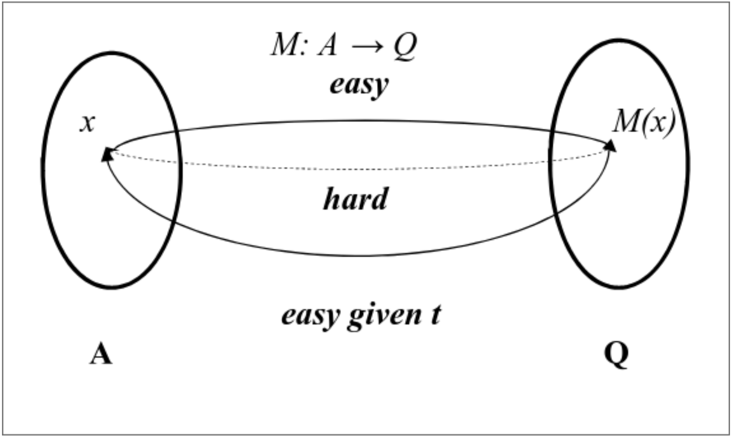


Figure: trapdoor function: eg. prime factor

Construct 1-out-of-2 OT by trapdoor function

1. A: $(f_1, t_1), (f_2, t_2) \leftarrow G$
2. B: $key \leftarrow G$
3. B: send $C = f_s(key)$
4. A: $key_1 = f_1^{-1}(C, t_1), key_2 = f_2^{-1}(C, t_2)$
5. A: $c_1 = Enc(m_1, key_1), c_2 = Enc(m_2, key_2)$
6. B: $m_s = Dec(c_s, key)$

It is easy to demonstrate that the construction is easily expandable to 1-out-of-N OT.

Expand & Disclaimer

1. The existence of such one-way functions is still an open conjecture. (P=NP)
2. A more general primitive for solving SMPC problem is GC (garbled curcuit)[GMW19, Yao82]
3. By the way, W in GMW is Avi Wigderson (Turing Award, 2023).
4. The best results on the fair coin-flip problem come from the [MNS09].
5. You could find the detailed and rigorous proof of security of Yao's protocol and GMW's protocol from [AL17] and [LP09].
6. For an overview of SMPC, see Lindell's survey[Lin20].

Applications

Secure Statistical Analysis

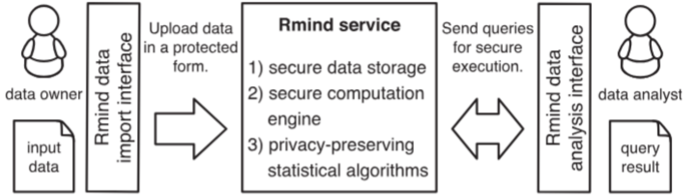

data owner
input data
Rmind data import interface
Upload data in a protected form.
Rmind service
1) secure data storage
2) secure computation engine
3) privacy-preserving statistical algorithms
Send queries for secure execution.
Rmind data analysis interface
data analyst
query result

Fig. 1. The main components and interfaces to RMIND.

Figure: Rmind: A Tool for Cryptographically Secure Statistical Analysis[BKLS16]

Applications

Federated Learning

1. Classic federated learning[MMR+17] use non-cryptographic security method like FedAvg.
2. MiniONN[LJLA17] use AHE to finish linear computation, and secret sharing & garbled curcuit to do nonlinear computation.
3. CryptoNets[GBDL+16] use leveled FHE for all computation.

Applications

Federated Learning

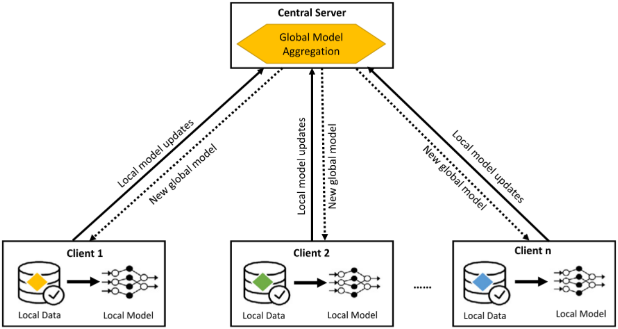


Fig. 1. Central federated learning.

Figure: Picture references: [KWL^{+}22]

Applications

PMT for Password Reuse Detection

PMT: Private Membership Test

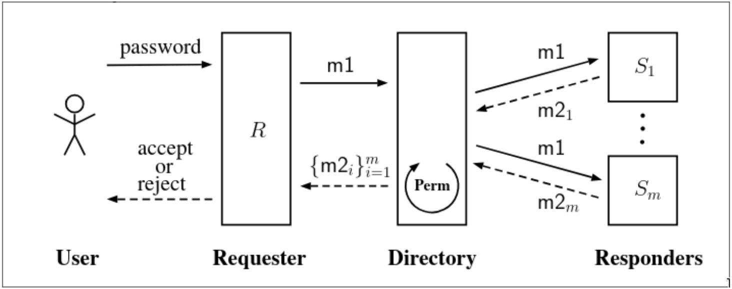


Figure: Architecture of the framework shown in [WR18]

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Thanks for Listening!